Methodological Deflationism and Metaphysical Grounding: From *Because* via *Truth* to *Ground*.

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Abstract

The paper proposes a strategy for understanding metaphysical grounding in deflationary terms and, more generally, proposes a form of methodological deflationism with respect to the notions of ground. The idea is to define a deflationary *is grounded in*-predicate by appeal to the two-place non-causal connective 'because' and a deflationary truth predicate. To this end we discuss the explanatory role of the truth-predicate in non-causal explanations and develop a theory of truth for the language of the 'because'-connective. We argue that at least from a logical perspective our deflationary notion of ground is up to the task.

1 Introduction

In recent years there has been a substantial amount of work on the notion of ground and the idea of metaphysical priority. The basic underlying idea of this work is that the relation of grounding orders its relata according to their metaphysical priority. For example, according to (1) the existence of the parts of a whole is metaphysically prior to the existence of the whole.

(1) The existence of a whole *is grounded in* the existence of its parts.

The interest in the notion of ground and the relation of grounding has been closely related to the increased interest in metaphysical or, more generally, non-causal explanations.[1] Indeed, the notion of ground and the relation of grounding are often thought to be for non-causal explanations what causality is for causal explanations: while in causal explanations explanans and explanandum are connected via some causal relation or mechanism, the explanandum will be grounded in the explanans in the case of non-causal explanations—the explanans is in some sense metaphysically prior.

While the ideas of grounding and metaphysical priority have attracted many they have also been met with a substantial amount of skepticism. In particular, Hofweber (2009) has accused

[1] See, for example, Fine (2012) for a discussion of the connection between grounding and metaphysical explanation and Poggiolesi and Genco (2021) for a more recent discussion on the connection between conceptual grounding and conceptual explanations.

proponents of the notion of ground of indulging in what he calls *esoteric* metaphysics. In rough terms, Hofweber argues that terms such as 'ground' and 'metaphysical priority' are accessible and understandable only to those already on board with these notions. According to Hofweber, the examples that are supposedly meant to provide us with a grasp of the notion of ground and related concepts fail to deliver the distinctive metaphysical sense the grounding-theorist is after, that is, these examples fail to provide us with an idea or understanding of the substantial metaphysical relation which is supposed to tie explanans and explanandum together. This puts the entire research program into question and, as a matter of fact, whilst not explicitly subscribing to Hofweber's argument, the underlying skepticism has been widely shared in the philosophical community. This leads to an unsatisfactory situation since the grounding-theorist will simply deny the charge and hold that ground and metaphysical priority are clear and accessible notions. As a consequence there will be no productive debate between the two camps.[2]

In this article we take another look at the skeptical stance towards the notion of ground and propose to reconstruct the metaphysician's theorizing in more transparent terms that should be acceptable to the skeptic. By rephrasing the debate we hope to provide a more productive analysis of the disagreement between the two camps and attempt to tie the underlying disagreement to a longstanding philosophical debate, namely, the debate between substantial and deflationary truth. The guiding idea of our reconstruction is to understand the metaphysician's *is grounded in*-predicate by appeal to the because connective, as it is used in non-causal explanations, and the truth predicate. To this end it is important to notice that non-causal explanations are not only employed in metaphysics, but also play an important role in other areas of philosophy as well as language, mathematics, and even science.[3] Of course, if non-causal explanations were confined to the domain of metaphysics, then Hofweber's skepticism would arguably affect the uses of the because connective in non-causal, i.e.,—in this particular case—metaphysical explanations. Non-causal explanations in mathematics, science and, to an important extent, language will not be affected by this skepticism and by appealing to these uses of the because connective in reconstructing the *is grounded in*-predicate such skeptical worries are avoided.

We start our investigation by some stage-setting and by proposing a definition of the metaphysician's *is grounded in*-predicate in terms of the truth predicate and the because connective (Section 2). The basic idea is to understand 'x *is grounded in* y' in terms of 'x is true because y is true'. Throughout the study we adopt a form of methodological deflationism towards the notion of truth and as a consequence, we argue, towards the notion of grounds. We take it that on this deflationary reconstruction the *is grounded in*-predicate is acceptable to the grounding-skeptic, but it is of course an open question whether the proposed reconstruction of the *is grounded in*-predicate yields a notion of grounds that is acceptable to the metaphysician. The bulk of the paper is devoted to this question. We tentatively argue that, at least from a logical perspective, the *is grounded in*-predicate we have defined yields an adequate notion of ground.

[2]Of course, Hofweber is not the only grounding skeptic and Hofweber's grounding-skepticism is not the only kind of skepticism. See Koslicki (2020) for an overview and discussion of various forms of grounding-skepticism. For example, grounding skeptics such as Wilson (2014) and Koslicki (2015) question whether grounding talk captures a single, unifying grounding-relation and hence whether grounding-talk is theoretically useful.

[3]See, e.g., Lange (2016) for a recent discussion of non-causal explanations in science.

To establish our conclusion we discuss the logic of the because connective in non-causal explanations (Section 3) and develop a theory of truth for the hyperintensional language of the because connective (Section 4). This leads to a discussion of the explanatory role of the deflationary truth predicate in non-causal explanations. Incidentally, this is a topic that awaits a systematic treatment in the literature despite the fact that it seems important to a number of deflationary proposals and, in particular, to Horwich's (1998b) *Minimalism*. We then show that against the backdrop of the theory of truth we developed our *is grounded in*-predicate satisfies precisely the logical laws of the metaphysician's ground-predicate (Section 5), that is, from a logical perspective the *is grounded in*-predicate is up to the job. However, the *is grounded in*-predicate gives rise to a partial notion of ground while the metaphysician is arguably also interested in the full and immediate notion of ground. The remainder of Section 5 investigates whether our proposal can be extended to the notion of full and immediate ground. In a nutshell, we argue that this will depend on the idealizations and abstractions of the because connective one deems acceptable and justifiable on the basis of our use of the because connective in non-causal explanations.

2 From *Because* via *Truth* to *Ground*

The strategy we use in defining the *is grounded in*-predicate on the basis of the truth predicate and the because connective is reminiscent of a trick Quine introduced in his attempted *Flight from Intensions* in relation to propositional attitude reports (Quine, 1956). Quine proposed to dispense of intensions by turning belief contexts into quotational contexts by appeal to a disquotational truth predicate. According to Quine the logical form of

(2) Mary believes that Kangaroos are dangerous.

should be understood as

(3) Mary believes-true 'Kangaroos are dangerous.'

In this paper we have no quarrel with intensions, but the de-nominalizing function of the truth predicate that Quine exploited will also be essential to our proposal. As in Quine's proposal, we want to construct, i.e. define, a predicate of sentences or—in our case—propositions on the basis of a sentential operator. The truth predicate enables us to transform arguments of the former into arguments of the latter. The sentential operator at stake is the two-place explanatory connective 'because' as it is used in non-causal explanations. As Fine remarks:

> "Perhaps the closest we come to an ordinary language formulation [of the notion of ground, JS] is with "because"."[4] (Fine, 2012, p. 46)

We propose to take this remark seriously and investigate how close we can get to the notion of ground by focusing on 'because' as it is used in non-causal explanations. However, as our previous remarks should have made clear, syntactically *because*, in contrast to *is grounded in*,

[4]In the same paragraph Fine moves on to argue that " 'because' does not convey the distinct sense of *grounds*". We discuss this more critical stance in Section 5.

takes sentences as arguments while the arguments of the *is grounded in*-predicate are, at least grammatically, phrases such as noun, determiner or complementizer phrases that can occupy nominal positions in a sentence. The truth predicate allows us to bridge this gap between the two grammatical categories in a systematic way by somewhat following the Quinean strategy, that is, we will understand grounding claims such as

(4) The fact that the ball is round and red *is grounded in* the fact that the ball is round.[5]

as

(5) It *is true* that the ball is round and red *because* it *is true* that the ball is round. [6]

More generally, we propose to define the *is grounded in*-predicate in the following way:

(DefG) x is grounded in $y =_{\text{def}} x$ is true because y is true.

If (DefG) is accepted, then, it seems, we have a perfectly clear and transparent way of understanding the notion of ground.[7] Moreover, to some extent (DefG) even explains the distinctive metaphysical sense of grounds: it is due to the *correspondence*-intuition commonly associated with the truth predicate, i.e., the intuition that statements or propositions are true due to the nature of reality. For even if one assumes the truth predicate to be merely an expressive device and, at best, to express an insubstantial property, one can acknowledge the initial plausibility of the intuition. If (DefG) is accepted, the discussion between grounding-skeptic and grounding-proponent will be on how serious we are to take the *correspondence*-intuition.[8] The grounding-theorist wedded to the distinctive metaphysical sense of the grounding-relation—or, perhaps, its metaphysical reality—will, arguably, adopt a full-blown correspondence theoretic account of truth and, accordingly, hold that because connective tracks a salient metaphysical ordering. The skeptic would arguably adopt a more deflationary theory of truth and deny that the because connective tracks any specific metaphysical ordering.

Of course, it remains to be argued that (DefG) yields a predicate of ground that is acceptable to the grounding-theorist. In other words, it needs to be shown that the *is grounded in*-predicate

[5]At this point we assume a relation of partial ground, that is, a relation that accounts for partial metaphysical explanations. Later in this paper we discuss the relation of full ground.

[6]A number of philosophers would argue that in (5) we are not appealing to the truth predicate but a truth operator 'it is true that'. However, as, e.g, Parsons (1993) points out, this view is in conflict with basically all contemporary theories of syntax, which interpret the that-clause as a unit. Of course, one can still argue that on the semantic level 'it is true that' should be treated as a unit, i.e. as an operator, but such a view requires a substantial argument and should not be considered as the default position. See also Stern (2016) for discussion. Similar remarks apply to the construction 'it is a fact that' used in Footnote 10.

[7]Admittedly, (DefG) gives unintended results if combined with explicit names of facts. For example, if we apply the definition (4) amounts to

($\star$) The fact that the ball is round and red is true because the fact that the ball is round is true.

which does not read well. However, for our proposal it is not necessary that ($\star$) is an acceptable reformulation of (4). Our claim is that (4) can be understood in terms of, that is reduced to, (5). Moreover, if some form of correspondence theory is assumed the *is grounded in*-predicate defined in (DefG) may well be understood as expressing a relation between facts. See the end of this section (Section 2) for some more remarks along these lines.

[8]See, for instance, Horwich (1998b, 2010) for a discussion of the *correspondence*-intuition and deflationism.

can satisfy the theoretical role of the metaphysician's grounding-predicate. This will depend on both the account of truth we adopt but also the account of the 'because'-connective we employ. For sake of the argument let us assume that we have a satisfactory account of the 'because'-connective at our disposal. Then whether (DefG) yields a predicate of ground that is acceptable to the metaphysician will depend on the theory and conception of truth we adopt. In turn, the philosophical moral of the reconstruction will vary depending upon the conception of truth at play. Suppose the theoretical role of the metaphysician's grounding-predicate requires a (substantial) correspondence theory of truth, then we would have a clear analysis of the distinct metaphysical sense the grounding-theorist alludes to. If, in contrast, it turned out that a deflationary account of truth yields an *is grounded in*-predicate that has all the important properties of the metaphysician's ground-predicate, then one might worry that the alleged metaphysical reality of grounding depends solely on the elusive *correspondence*-intuition the deflationist hopes to account for in insubstantial terms. Arguably, in this case the distinct metaphysical sense of grounding needs to be located elsewhere, if the deflationary account of truth is deemed viable.

In this paper, our strategy will be to entertain a deflationary perspective and assume, somewhat following the outlines of Field (1994), a form of *methodological deflationism* towards the notion of truth and, as we shall see, the notion of ground. The idea is to adopt a deflated account of ground as a working hypothesis and investigate whether more substantial metaphysical assumptions are needed and, if so, at which point these assumptions are doing actual philosophical work for the grounding-theorist. If it turned out that these assumptions are philosophically indispensable while, at the same time, theoretically motivated, it seems that the skeptical case would be substantially weakened. However, if, on the contrary, the theoretical role of the notion of ground could be fully captured using a deflationary *is grounded in*-predicate, then there would seem be some pressure on the grounding-theorist to pinpoint the precise source of the alleged metaphysical sense of the notion of ground to counter the charge of indulging in esoteric metaphysics. Of course the most likely outcome is that neither conclusion can be firmly established. Rather, the moral of the investigation will depend on one's view of grounding and there is bound to be substantial disagreement between grounding-skeptic and grounding-theorist. However, we hope that in the present setting a more constructive account of their disagreement can be given that leads to a fruitful philosophical debate rather than a mere clash of intuitions. Adopting this spirit, we remain neutral throughout the paper as to whether the full theoretical role of the notion of ground can be accounted for on the basis of our deflationary understanding of ground. Yet, we argue that the deflationary proposal should not be dismissed out of hand. To this effect we show that at least with respect to the *logical role* the deflationary notion of ground is up to the job, that is, the deflationary *is grounded in*-predicate is characterized precisely by the logical laws of the metaphysician's ground-predicate.

Before we can turn to establishing the latter claim, we need to take a closer look at the 'because'-connective and its interpretation. The definiens of (DefG) does not only rely on the truth-predicate but crucially also uses the two-place explanatory connective 'because'. In light of this a metaphysician may hold that the proposed methodologocial deflationism is up to a wrong start since the metaphysical import is via the because connective rather than via the truth predicate: while the truth predicate may well be a deflationary truth predicate, the *is* *grounded in*-predicate will not be deflationary since we introduce metaphysical assumption via the because connective. On this view, an adequate account of the because connective in non-causal explanations depends on inflationary assumptions. For example, one may think that our understanding of the because connective in non-causal explanations presupposes some non-deflationary relation of ground since the latter will be crucial for providing a semantic interpretation of 'because', that is, the meaning of 'because'. We will resist this idea and assume our understanding of the (non-causal) because connective to be determined by our use of the because connective within non-causal explanations in the sense of Horwich (1998a), that is, by "the regularities governing our deployment of the sentences in which it [the word , JS] appears" (Horwich, 1998a, pp. 2/3) within non-causal explanations. According to Horwich these regularities of use are due to the specific acceptance property associated with 'because', that is, the conditions that stipulate when a sentence containing 'because' is accepted (cf. Horwich, 1998a, Ch. 3). While the particulars of such an account may be in need of further clarification, it does not seem to be an unreasonable view that should be dismissed out of hand. More to the point, the view is not necessarily less plausible than the view that understanding our uses of 'because' requires stipulating a grounding-relation or some other substantial assumption. Moreover, it is worth recalling that we are entertaining (DefG) and the deflationary account of ground as a methodological assumption: perhaps a use-theoretic characterization of the because connective will prove unsatisfactory and, as a consequence, the deflationary account of ground is ultimately bound to fail, but this is precisely one of the questions at stake.[9]

Finally, one further preliminary remark concerning our proposed understanding of (4) in terms of (5) seems in order: at first glance, it may seem that a deflationary *is grounded in*-predicate cannot be the one figuring in (4) above since, in (4), 'is grounded in' applies to facts and this clearly undermines the idea of ground qua deflationary notion. If ground-theoretic deflationism is worth its name it should not allow the import of substantial assumptions by stipulating grounds to be facts or similar entities. However, arguing against the proposed definition by appeal to example (4) seems, again, premature: it is precisely the question at stake whether the *is grounded in*-predicate can be understood in a deflationary way and whether the appeal to facts in sentences such as (4) needs to be understood substantially.[10] In this article our official stance is to assume grounds to be propositions of the kind appealed to by Horwich (1998b) although we wish to leave this choice somewhat open: for, depending on the particular truth theory one adopts, the *is grounded in*-predicate will apply to objects of different type:[11]

[9]There is also a somewhat weaker and less ambitious understanding of the proposed methodological deflationism. The project may be understood to be directed merely at the metaphysical realist's understanding of *is grounded in*. On this view one can allow for an inflationary understanding of 'because', as long as it does not derive from the postulation of a substantial (mind-independent) explanatory relation. For example, if our understanding of 'because' is tied to its inferential role in the sense of proof-theoretic semantics (cf., e.g., Schroeder-Heister, 2018), then *is grounded in* will not be deflationary in the strict sense. Nonetheless, assuming a deflationary truth predicate the proposed reconstruction could still be used to explain how the distinct metaphysical sense of grounding, i.e., its alleged metaphysical reality, arises along the outlines sketched above.

[10]The question is whether the notion of fact figuring in (4) is fully captured by the deflationary schema

$$\Phi \leftrightarrow \text{It is a fact that } \Phi$$

or whether further, substantial assumptions are required.

[11]For sake of this paper we assume with Horwich that a deflationary acceptable notion of proposition is available. For our purpose it is important that propositions are of similar grain as purely syntactic objects, that is,

our proposal reduces the question of the appropriate relata of the grounding-relation to the well-discussed question of the appropriate bearers of truth.[12]

With this out of our way let us focus on the task at hand and investigate the logical properties of the *is grounded in*-predicate. To kick things off we discuss the logic of the because connective that is intended to capture the uses of 'because' within non-causal explanations.

3 Because

In natural language 'because' connects two statements to form a new statement akin connectives such as 'and' or 'or'. The because connective figures prominently within putative explanations such as

(6) The window broke because a stone was thrown at it.

(7) The water is boiling because it was heated to 100°C.

(8) The mother failed to distribute the 23 strawberries evenly among her three children without slicing because 23 cannot be divided evenly by 3.[13]

(9) The sum of the angles of any quadrangle is equal to 360° because the sum of any triangle is equal to 180°.[14]

(10) The pious is pious because it is loved by the gods.

(11) There's a table because there are simples arranged tablewise here.

These explanations consist of an explanandum—the statement on the left of the 'because'-connective—and an explanans—the statement on the right. The examples suggest that 'because' can be used to provide causal as well as non-causal explanations: (6) and (7) are clearly causal explanations, while at least prima facie (8), (9), (10) and (11) are non-causal.[15] Moreover, at least (8) and (9) should be acceptable to the Hofweber-style skeptic and should be acceptable corpora for our deflationary project. We focus on such uses of 'because' in non-causal explanations and assume that they indeed amount to non-causal uses. The uses we are mostly interested in will

sentences. This enables the use of the common technical machinery in working out the formal details of our account.

[12]Tying grounds and truths together points to an alternative way of conceiving of (DefG) that might be of greater appeal to the grounding-theorist. If, in contrast to our proposal, the because-operator is given a substantial reading, (DefG) may be thought to bridge the gap between so-called *predicational* and *operational* views of grounding: the predicational view respects surface grammar and conceives of grounding as a predicate while the operational view conceives of grounding via a sentential connective similar to the because-operator. Proponents of the predicational view include Schaffer (2009) and Rosen (2010) while the operator approach is championed, e.g., by Fine (2012) and Correia (2010).

[13]Adapted from (Lange, 2016, p. 6).

[14]Cf. Bolzano (2015).Thanks to an anonymous referee for helping me to improve on a previous example.

[15]We follow Lange (2016) in understanding 8 as an example of a non-causal explanation. The idea is that it is the mathematical fact that is doing the explanatory work in this case.

be those in theoretical and scientific contexts that presuppose theoretical, philosophical, or linguistic reflection. We take it that most examples in support of non-causal uses of 'because' are heavily theory-laden as can be witnessed by (11) above. Indeed, most examples brought forward by the most detailed linguistic investigation of non-causal uses of 'because' by Shaheen (2017) are taken from philosophical or scientific corpora rather than ordinary discourse.[16] It is the uses of 'because' in such theoretical, non-causal contexts that we aim to characterize in this section.

Fortunately, there is extant work by Schnieder (2011) that fits the bill. Schnieder (2011) proposes a logic of 'because' focusing on the non-causal, explanatory uses.[17] In this paper we adopt Schnieder's logic. While certain details of our proposal may be specific to this particular logic its general outline should apply to alternative characterizations of 'because'.[18] Schnieder takes the because connective to express a *partial* explanatory connection between the arguments of the connective. Confining oneself to partial explanations is mostly justified by analogy to causal explanations and the use of 'because' in such contexts: it is a common feature of such explanations that they are partial as can be witnessed by statements such as (7) for which we need to supplement the explanans by supplementary information, for example, that water is heated at standard atmospheric pressure.

Schnieder's logic is formulated in a standard first-order language and assumes classical logic. In our case we will work in the specific first-order language $\mathcal{L}_{\mathrm{T}}$ which contains a specific one-place predicate constant T, namely, the truth predicate. Schnieder proposes four structural assumptions concerning the because connective in non-causal explanations: the connective is irreflexive, transitive, and factive. This leads to the following four axioms of the logic of 'because'.[19] Perhaps slightly confusingly we denote 'B because A' by '$A \prec B$' (read: that A explains that B).

(IR) $\neg(A \prec A)$

(Trans) $(A \prec B) \longrightarrow ((B \prec C) \longrightarrow (A \prec C))$

(FactL) $(A \prec B) \longrightarrow A$

(FactR) $(A \prec B) \longrightarrow B$

All of these structural assumptions have been question in the literature on grounding to varying degrees, but we will not enter these discussions. The remaining axioms of Schnieder's logic

[16]Shaheen (2017) argues that 'because' is lexically ambiguous and polysemous. According to Shaheen (2017) there are two closely related senses of 'because': one roughly covering causal explanations and the other one covering metaphysical explanations. Since Shaheen (2017) uses causal explanations as an opposite to metaphysical explanations, it seems reasonable to take the term 'metaphysical explanation' to stand for non-causal explanations more generally. This also seems to match the examples used by Shaheen (2017).

[17]Schnieder's logic is very much related to the logic of ground proposed by Fine (2012) or Correia (2010), but in contrast to the aforementioned authors he explicitly aims at describing the non-causal, explanatory uses of 'because' in natural language. Even though Schnieder (2011) does not explicitly confine the scope of the logic to the sort of theoretical uses of 'because' we alluded to, we think that Schnieder would be happy to accept this restriction—indeed we think he must accept this restriction since he is otherwise vulnerable to the kind of objections made in Tsohatzidis (2015). See Schnieder (2016) for a reply to Tsohatzidis.

[18]See, e.g., Poggiolesi (2020) for a recent overview of different logics.

[19]We diverge from Schnieder's (2011) formulation by presenting the logic in axiomatic form rather than as a natural deduction system.

characterize the interaction of the because connective with the truth-functional connectives. To this effect Schnieder relies on the following *Core Intuition*:

> "A sentence governed by a classical truth-functional connective has its truth-value *because of* the truth values of the embedded sentences." (Schnieder, 2011, p. 448)

The intuition can be used to determine axioms for all truth-functional connectives and, albeit indirectly, the quantifiers. For example, for disjunction we would have axioms such as

$(<\vee 1)$ $\quad A \longrightarrow (A < (A \vee B))$

$(<\vee 2)$ $\quad B \longrightarrow (B < (A \vee B))$

and for the negative case

$(<\vee 3)$ $\quad \neg A \wedge \neg B \longrightarrow (\neg A < \neg(A \vee B))$

$(<\vee 4)$ $\quad \neg A \wedge \neg B \longrightarrow (\neg B < \neg(A \vee B))$.

For each primitive binary connective we thus need a total of four axioms. To avoid a painful long list of axioms we therefore remain somewhat restrictive with respect to our choice of primitives and limit ourselves to negation $\neg$, conjunction $\wedge$, and the universal quantifier. The remaining connectives and the existential quantifier are considered to be defined in the sense that they are mere notational abbreviation of their usual definiens.[20]

$(<\neg)$ $\quad A \longrightarrow (A < \neg\neg A)$

$(<\wedge 1)$ $\quad A \wedge B \longrightarrow (A < (A \wedge B))$

$(<\wedge 2)$ $\quad A \wedge B \longrightarrow (B < (A \wedge B))$

$(<\wedge 3)$ $\quad \neg A \longrightarrow (\neg A < \neg(A \wedge B))$

$(<\wedge 4)$ $\quad \neg B \longrightarrow (\neg B < \neg(A \wedge B))$

$(<\forall 1)$ $\quad \forall x A \longrightarrow (A(t) < \forall x A)$

$(<\forall 2)$ $\quad \neg A(t) \longrightarrow (\neg A(t) < \neg\forall x A)$[21]

In judging the plausibility of the axioms it is important to appreciate that we are focusing on the theoretical uses of 'because', that is, uses which presuppose a certain amount of theoretical reflection. Otherwise these axioms seem implausible as was pointed out by Tsohatzidis (2015).

(12) ??If Tom is alive then he is alive or he is dead because he is alive.[22]

[20] On this account the Boolean laws are instances of the definition and in particular formulas $\neg(A \wedge B)$ and $\neg A \vee \neg B$ are actually one and the same formula (and thus explanatorily equivalent). This may seem unfortunate from a philosophical point of view. However, nothing hinges on our restrictive choice of primitives. We could have more primitives and block this unwelcome consequence, yet we would end up writing down long lists of principles, which we want to avoid for reasons of efficiency.

[21] $A, B, C, \ldots$ are schematic variables for formulas of the language including the truth predicate ($\mathcal{L}_{\mathrm{T}}$). The axioms are to be understood as universal closures.

(12), at least on the face of it, seems odd if it is taken to be part of ordinary discourse. But now compare

(13) If 2+2=4, then 2+2=4 or 3+3=5 because 2+2=4.

in a context where we try to explain an admittedly simplistic mathematical proof. In this theoretical context (13) seems absolutely fine. Similarly, (12) seems acceptable, if we imagine a context in which we explain a simplistic philosophical argument to an interlocutor. So focusing on these theoretical uses of 'because' we take it that the axioms aptly characterize the because connective in non-causal (partial) explanations.

A noteworthy consequence of this characterization of the because connective is that it is a hyperintensional connective, that is, co-intensional formulas cannot be substituted for each other in the scope of the because connective.[23] Indeed the because connective must be hyperintensional, for otherwise this would contradict the axiom (IR), that is, the irreflexivity of the because connective.[24] As a matter of fact, the logic does not license *any* substitutions within the scope of the because connective and this will cause a number of complications when we introduce a truth predicate to the language. Ultimately, this will lead to the introduction of so-called *Substitution axioms*: axioms that license the substitution of certain sentences for each other in the scope of the because connective.

4 Because and Truth

Methodological deflationism concerning the notion of ground will only lead to an interesting proposal if the truth predicate employed in the definiens of the *is grounded in*-predicate, together with its truth theory, does not appeal to assumptions, which are unacceptable from a deflationary point of view. The challenge then is to provide an account of deflationary truth in non-causal explanations, that is, we need to provide a precise account of the interaction of the deflationary truth predicate and the non-causal because connective. There are a plethora of different versions of truth-theoretic deflationism and to some extent we will remain agnostic with respect to the specific deflationary truth theory at play.[25] But we take one unifying feature, and central characteristic, of the deflationary truth predicate to be the idea that it is merely an expressive device (Quine, 1970; Horwich, 1998b; Field, 1994). It is required for expressing infinite conjunctions and disjunctions and, more generally, that it is a device for performing semantic ascent and descent. Crucially, on this view the truth predicate does not play an explanatory

[22] The example is modified from (Tsohatzidis, 2015, p. 47). In its original formulation it was directed against Schnieder's (2011) natural deduction system for the logic of 'because'. In this case it is even more compelling:

Tom is alive. He is alive or he is dead because he is alive.

[23] See Berto and Nolan (2021) for more on hyperintensional context and hyperintensionality more generally.

[24] We have $A \vee \neg A < (A \vee \neg A) \vee B$. But $A \vee \neg A$ and $(A \vee \neg A) \vee B$ are logically equivalent , and if $<$ were not hyperintensional we could infer $A \vee \neg A < A \vee \neg A$, which contradicts (IR).

[25] We also put the problems of truth-theoretic deflationism aside and, for the sake of this paper, assume that it is a coherent position. Problematically, we also lump linguistic, conceptual, and metaphysical deflationism (cf. Bar-On and Simmons, 2007) together for the sake of this paper. Differentiating between the different forms will lead to quite a variety of different deflationary views of ground.

role in causal explanations or explanations that are substantial in the metaphysical sense. As a consequence, if we were interested in ‘because’, as it is used in *causal* explanation, and its interaction with a deflationary truth predicate, we should be able to introduce and eliminate the truth predicate, that is, quote and disquote in the scope of the because connective without altering the truth value of the explanation. Since we are dealing with ‘because’ in non-causal explanations however, the deflationary truth predicate may well have an explanatory role to play in these contexts. For example, Horwich (1998b, 2010) holds that within non-causal explanations governed by the because connective the truth predicate is *not* explanatorily innocuous. More precisely, he claims that

> “<Snow is white> is true because snow is white.”[26] (Horwich, 1998b, p. 105)

Thus, Horwich holds that “<Snow is white>’s being true is explained by snow’s being white.” (Horwich, 1998b, p. 104) and as a consequence the two sentences ‘Snow is white.’ and ‘<Snow is white> is true.’ are not explanatorily equivalent—the truth predicate is not explanatorily innocuous.

Pinning down the explanatory role of the truth predicate in non-causal explanations is one of the principal challenges of introducing the truth predicate to a language containing the because connective. Surprisingly, we thereby enter into hitherto unexplored territory: to our knowledge no theory of truth for a language with the non-causal because connective has been developed to date.[27] In what is to come we consider two different conceptions of deflationary truth in non-causal explanations and show that on both views the *is grounded in*-predicate we propose will have the logical properties of the grounding-predicate introduced by the grounding-theorists. The first view, labeled *Strongly Transparent Truth* will conceive of the truth predicate, contra Horwich, as being explanatorily innocuous, that is, the sentence ‘Snow is white.’ and the sentence ‘It is true that snow is white.’ will be considered as explanatorily equivalent. The alternative view, labeled *Aristotelian Truth* will follow the outlines of Horwich’s proposal. However, before we spell out these rival views we focus on their common core, that is, the aspects of the theory of truth that are shared by the two diverging views.

All theories of truth have to answer the paradoxes of truth and, in particular, the Liar paradox in one way or another. In a nutshell, there are three options how this can be done: First, one can restrict the salient, characteristic principles of truth to a paradox free fragment of the language. Second, one can reject the extant principles of truth in favor of weaker principles of truth that do not have paradoxical consequences. Thirdly, one can avoid paradox by adopting a non-classical logic. We opt for the first option and provide a typed theory of truth: we can truthfully say of a proposition (sentence) that it is true only if the proposition (sentence) does not appeal to the truth predicate. Ultimately, this may not be the most attractive way of dealing with the paradoxes but it is simple and suffices for our proposal. Assuming a typed framework also has the advantage that we have to compromise neither on the principles of truth we assume nor on the underlying logic. We can stick to our naive intuitions regarding truth within the setting of classical logic and, more generally, the logic of ‘because’. As we

[26] According to Horwich’s notation ‘<Snow is white>’ is a name of the proposition that snow is white.

[27] Some remarks in this direction may be found, e.g., Fine (2010) and Litland (2015). However, these works fall short of developing a precise theory of truth within the language of a hyperintensional explanatory connective.

pointed out, the deflationary truth predicate is meant to be an expressive device and, as a consequence, a device for performing semantic ascent and descent. This aspect of the truth predicate is aptly expressed by the so-called T-scheme (TS), which says that for all sentences Φ of the language *without* the truth predicate that it is true that Φ if and only if Φ:

(TS) $$\mathrm{T}\ulcorner \Phi \urcorner \leftrightarrow \Phi.$$[28]

The T-scheme is the characteristic principle of truth and is at the center of most deflationist accounts of truth. Indeed deflationists like Horwich (1998b), but also Field (1994), will hold that (TS) fully characterizes the notion of truth and that it suffices to account for the expressive function of the truth predicate they hold dear. But, as has been pointed out by, e.g., Gupta (1993b,a), without further assumptions the T-scheme will not suffice for deriving generalizations such as

(14) There exists no sentence such that the sentence and its negation are true.

Moreover, for the same reason it will be impossible to derive that all instances of the logical axioms are true, within a deflationist account based solely on the T-scheme.[29] The ability to express such generalizations was thought to be one of the main characteristics of the deflationary truth predicate and a deflationary theory of truth which misses out on such general claims is thus clearly unsatisfactory. As a consequence, theories of truth have been based on so-called compositional principles of truth, such as (15),

(15) A conjunction is true, if and only if both its conjuncts are true.

that characterize the interaction of the truth predicate with the logical connectives. It remains an ongoing discussion whether deflationists that take the T-scheme to be the characteristic principle of truth are licensed to assume these compositional principles or whether the compositional principles add further theoretical commitments to the theory (Field, 1994, 2006; Heck, 2021, 2018). We put this discussion aside and grant the deflationist the appeal to compositional principles without further justification. Indeed, we shall assume a number of compositional principles as the basic axioms of our truth theory.

4.1 The Theory of Compositional Truth

The theory of compositional truth is based on the aforementioned compositional principles and assumes classical logic. It is formulated in the language $\mathcal{L}_{\mathrm{T}}$, which extends the language $\mathcal{L}$ by a one-place predicate constant T—the truth predicate. $\mathcal{L}$ contains the $<$-connective, which we assume to be governed by Schnieder's logic of because, and extends the language of some

[28] $\ulcorner \cdot \urcorner$ is a name forming device which applied to a sentence yields, e.g., the name of the proposition expressed by Φ, the name of the sentence Φ, or the name of some alternative suitable bearer of truth that is deflationary acceptable. We use $\Phi, \Psi, \ldots$ as schematic letters for sentences of the language without the truth predicate; A, B, C are used as schematic letters for formulas of the entire language, i.e. the language with truth predicate, while φ, ψ, χ will be used as individual variables ranging over sentence-like truth bearers (see below).

[29] Based on the T-scheme we can of course show the truth of each individual instance of a logical axiom but we cannot derive the universal generalization.

syntax theory, e.g., the language of arithmetic. Besides the compositional axioms for truth discussed below the theory has axioms defining the basic operations of the syntax theory. For further specifics we refer the reader to Footnote 31 below and for more general background to Halbach (2014); Halbach and Leigh (2022).

The compositional axioms for truth specify how the truth predicate interacts with the logical connectives and quantifiers. It is a common theory when applied to extensional and, with some qualifications, to intensional languages but to our knowledge has not been applied to hyperintensional languages such as the language of the because connective.[30] In addition to the compositional axioms the theory requires that the scheme (TS) holds for atomic sentences of the language without the truth predicate. For sake of simplicity we assume that the language has just one predicate constant, namely, the identity symbol. As a consequence, (TS) for atomic formulas will be (T-At) below. The theory of compositional truth $\mathsf{CT}_{\prec}$ comprises the axioms

(T-At) $$\forall x, y(x = y \leftrightarrow \mathrm{T}[\dot{x} = \dot{y}]);$$

(T¬) $$\forall \varphi(\mathrm{T}[\neg\varphi] \leftrightarrow \neg\mathrm{T}[\varphi]);$$

(T∧) $$\forall \varphi, \psi(\mathrm{T}[\varphi \wedge \psi] \leftrightarrow \mathrm{T}[\varphi] \wedge \mathrm{T}[\psi]);$$

(T∀) $$\forall \varphi(v)(\mathrm{T}[\forall v \varphi(v)] \leftrightarrow \forall y \mathrm{T}[\varphi(\dot{y}/v)]).$$ [31]

This leaves us with the task of providing a truth-axiom that characterizes the interaction of the truth predicate and the because connective. We propose the following compositional axiom to complete the theory $\mathsf{CT}_{\prec}$:

(T≺) $$\forall \varphi, \psi\,(\mathrm{T}[\varphi \prec \psi] \leftrightarrow \mathrm{T}[\varphi] \prec \mathrm{T}[\psi])\,.$$

This axioms says that an explanation is true if and only if the explanandum is true because the explanans is true, that is, the axioms allows us to move the because connective in and out of the scope of the truth predicate. Let us first apply the axiom to a sample explanation, say (9). According to (T≺) the following two claims are equivalent:[32]

[30] See Halbach (2014) for a presentation of the theory and a discussion of some of its properties.

[31] As indicated in Footnote 28 we use φ, ψ, χ as individual variables ranging over sentences-like truth bearers, that is, objects that have sufficient structure so that the relevant syntactic operations can be defined for these objects. As far as the formulation of the formal theory is concerned we assume these objects to be sentences but philosophically we think of them as propositions, that is, the syntax theory can also be conceived of as a theory of structured propositions along the lines of King (2007). $\varphi(v)$ is an individual variable ranging over formulas with the free variable v. Rectangular brackets indicate the scope of the truth predicate; the logical connectives, quantifiers and the identity symbol within these brackets should be understood as their corresponding syntactic operations. This means that ,e.g., '$[\varphi \wedge \psi]$' will be the first-order term '$\varphi^\frown$'∧'$^\frown\psi$', which given a suitable assignment of values to the variables will denote a sentence of $\mathcal{L}$. As mentioned, we assume the theory extends some suitable syntax theory that suffices to define the usual syntactic notions. The function symbol $\dot{\eta}$ (where η designates the argument position) is the representation of, i.e. a name of, the function that takes expressions of the language (natural numbers in the case of arithmetics) as input and yields the canonical name of that expression (the Gödel number of its numeral) as an output. Such a function is needed to quantify into quotation contexts. For sake of simplicity we have omitted one axiom, which says that only sentences of the language without the truth predicate are true:

(Snt) $$\forall x(\mathrm{T}x \longrightarrow \mathsf{Sent}_{\mathcal{L}}(x)).$$

[32] To check whether (T≺) is backed by our natural language intuitions it would be preferable to use 'it is true that' instead of the quotational construction appealed to in (16). But, unfortunately, scope ambiguities arise. So we decided to stick to the quotational formulation.

(16) ‘The sum of the angles of any quadrangle is equal to 360° because the sum of any triangle is equal to 180°’ is true.

(17) It is true that the sum of the angles of any quadrangle is equal to 360° because it is true that the sum of any triangle is equal to 180°.

As far as our intuitions can be deemed trustworthy with respect to complicated sentences like (16) and (17), considering these sentences to be equivalent seems acceptable to our ears. Admittedly there might be some room for debate but we have been unable to come up with a clear counterexample to (T<).[33] Moreover, preempting the definition of the *is grounded in*-predicate, the axiom says that a because-statement is true, if and only if, the explanandum *is grounded in* the explanans, which seems to be a correct outcome.

To give further justification for (T<), we can turn to a more theoretical perspective. Ultimately, the acceptability of the principle hinges on the question of whether there is a difference in performing semantic ascent on the global or on the local level. In other words, given some explanation is there a difference between raising an explanation to the metatheoretical level by performing a semantic ascent on the explanation as a whole or by performing a semantic ascent on explanans and explanandum individually?

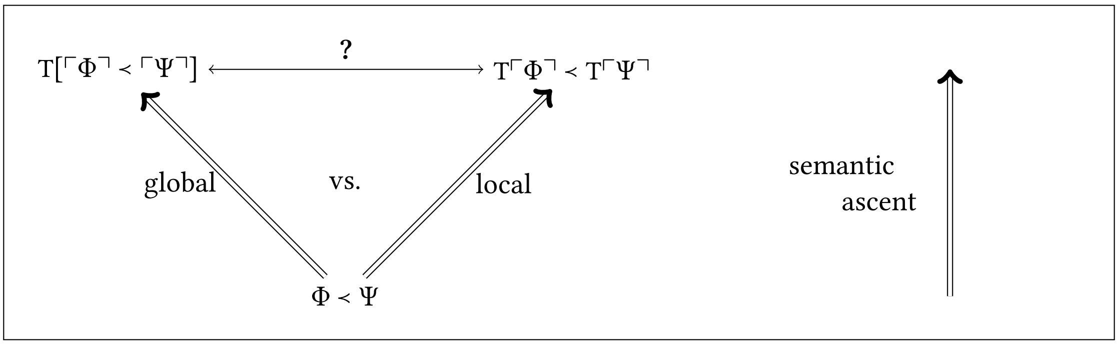

Figure 1: (T<) and semantic ascent

We take it that a deflationist should accept (T<) because, even though we are talking about non-causal and, in the metaphysical sense, insubstantial explanations, breaking the equivalence between $T\ulcorner\Phi < \Psi\urcorner$ and $T\ulcorner\Phi\urcorner < T\ulcorner\Psi\urcorner$ would require a convincing story why the allegedly innocent deflationary truth predicate has such an impact within non-causal explanations. Why should semantically raising explanans and explanandum simultaneously break the explanatory connection? In the absence of a convincing story to this effect we propose adopting (T<) as aptly

[33] If following Tsohatzidis (2015) we take (12) to be unacceptable, then counterexamples may be readily available. Let $*$ be the name of the because-statement displayed in (12). Then (T<) says that

$*$ is true if and only ‘Tom is dead or he is alive’ is true because ‘Tom is alive’ is true.

Now, somewhat convincingly Tsohatzidis (2015) argues that we judge the right-hand-side of the equivalence to be correct, which would yield a counterexample to (T<). However, we have ruled out an understanding that deems (12) unacceptable, so these examples are no threat to the plausibility of (T<) in the present context.

characterizing the interaction of the truth predicate and the because connective. Notice that accepting $(\mathrm{T}\prec)$ does not commit one to the view that the truth predicate is explanatorily innocuous in the sense that Φ and $\mathrm{T}\ulcorner\Phi\urcorner$ can be substituted *salva veritate* within the scope of the because connective. In particular, as we shall discuss in Section 4.3, one can still follow Horwich (1998b, 2010) to hold that Φ explains $\mathrm{T}\ulcorner\Phi\urcorner$. On this view, semantic ascent is explanatorily directed but it does not matter whether we perform it globally or locally.

4.1.1 An argument against $(\mathrm{T}\prec)$ based on the notion of full and immediate ground

Before we move on and discuss further axioms governing the interaction of the truth predicate and the because connective it may be worth considering an argument against $(\mathrm{T}\prec)$ some grounding-theorists may be eager to bring forward.[34] The argument purports to show that $(\mathrm{T}\prec)$ is in conflict with understanding the explanatory connective as expressing full and immediate explanatory connections and the idea that semantic ascent is explanatorily directed we have just alluded to.

Up to this point we have taken the because connective to express a partial explanatory connection. On this understanding the *is grounded in*-predicate to be defined will give rise to a notion of partial ground. However, grounding-theorists are also interested in the notion of full and immediate ground. More importantly, theorists such as Fine (2012) take the notion of full and immediate ground to be the fundamental notion of ground since, it seems, the notion of partial and mediate ground can be defined on the basis of the notion of full and immediate ground but not vice versa. As the name suggest, a full and immediate ground of a fact $\ulcorner\Phi\urcorner$, in contrast to a merely partial and mediate ground, fully grounds the fact $\ulcorner\Phi\urcorner$ and, moreover, the grounding should not be mediated via some other fact but directly.[35] In Section 5 we shall discuss whether our methodological deflationism is able to account for the notion of full and immediate ground in more general terms. However, some grounding-theorists may argue that we cannot because of the axiom $(\mathrm{T}\prec)$. The idea of the argument is that in order to stand a chance of defining a deflationary *is grounded in*-predicate we need to provide a theory of truth for a explanatory connective that expresses a full and immediate explanatory connection. But such an explanatory connective in combination with the view of Aristotelian Truth, i.e., the view that semantic ascent is explanatorily directed, seems to lead to a contradiction. As we shall discuss in more detail in Section 4.3,

$$(\prec\mathrm{T}\!\uparrow) \qquad \Phi \longrightarrow (\Phi \prec \mathrm{T}\ulcorner\Phi\urcorner).$$

is the constitutive principle of this view. The principle asserts that it is true that snow is white because—understood as a full and immediate explanatory connective—snow is white. Moreover, since the principle is thought to be the only axiom of the logic of because that yields an explanation for a sentence of the form $\mathrm{T}\ulcorner\Phi\urcorner$ it seems reasonable to hold that an explanans of $\mathrm{T}\ulcorner\Phi\urcorner$ is either explanatorily equivalent to Φ or Φ itself.[36]

[34] The argument against $(\mathrm{T}\prec)$ was first brought to my attention by Fabrice Correia.

[35] See Fine (2012) for a discussion of the notion of full and immediate ground.

[36] It is hard to find an explicit statement of this view in the literature. However, the idea has been put forward to me by a number of grounding-theorist in private communication and fits with assumption on grounding made by, e.g., Correia (2017); Poggiolesi (2016, 2018, 2022), and Wilhelm (2021).

But now assume Φ, Ψ and $\Psi \prec \Phi$. Then $T\ulcorner\Psi \prec \Phi\urcorner$ by (TS) and by (T≺) we infer $T\ulcorner\Psi\urcorner \prec T\ulcorner\Phi\urcorner$. But by (≺T↑), $\Phi \prec T\ulcorner\Phi\urcorner$, that is, we have two full and immediate explanations of $T\ulcorner\Phi\urcorner$: Φ and $T\ulcorner\Psi\urcorner$. It is not hard to find examples for which Φ and $T\ulcorner\Psi\urcorner$ cannot be consistently held to be explanatorily equivalent given our logic of because. Hence, if the initial assumptions are granted, we have derived a contradiction. Ultimately, this does not only suggest that there is a problem for extending our approach to the notion of full and immediate ground, but it also challenges our approach more generally since it undermines the theoretical justification we offered for (T≺): after all it does seem to make a difference from an explanatory perspective whether we perform semantic ascent globally or locally.

However, before jumping to this conclusion and blaming (T≺) for the unwelcome consequences, it is important to notice that Aristotelian Truth, that is (≺ T ↑), has some odd and counterintuitive consequences independently of (T≺), if the explanatory-connective is meant to express a full and immediate explanatory connection: on this understanding (≺T↑) implies that there is no, i.e., not even a partial, mediate explanatory connection between $T\ulcorner\Phi\urcorner$ and $T\ulcorner\Phi \wedge \Psi\urcorner$.[37] This seems very odd to say the least. The problem is that by assumption $\Phi \wedge \Psi$ fully and immediately explains $T\ulcorner\Phi \wedge \Psi\urcorner$ and hence $T\ulcorner\Phi\urcorner$ cannot play a role in the explanation of $T\ulcorner\Phi \wedge \Psi\urcorner$. But it seems equally implausible to hold that $T\ulcorner\Phi \wedge \Psi\urcorner$ explains $T\ulcorner\Phi\urcorner$, as this would suggest that the truth of a conjunction would explain the truth of its conjuncts.[38] In conclusion there seems to be a problem with combining the idea of Aristotelian Truth and of a full and immediate explanation (or ground), which is independent of the axiom (T≺).

Upon reflection it seems that the problems we have encountered stem from the fact that according to the view of Aristotelean Truth semantic ascent is explanatorily directed. This introduces a new direction of explanation, that is, a vertical direction in addition to the horizontal direction of explanation that the basic axioms of the logic of the because connective aim to capture. But if we wish to allow for reasonable horizontal explanations between propositions of the same semantic level at every—not just the base—level, then there will be problems if we apply the idea of full and immediate explanations in a non-discriminatory fashion. This shows, we take it, that we must distinguish between horizontal and vertical full and immediate explanations. But then there is no conceptual problem in allowing for two full and immediate explanations: a horizontal and a vertical one. The proponent of Aristotelian Truth should be happy with this proposal as it blocks the troublesome consequences of their view without discrediting the idea of a full and immediate explanation. One may even say that in those cases where we have two conflicting full and immediate explanations the vertical explanation is the more fundamental one.[39] However, on this view the argument against (T≺) needs to be rejected.

[37]Here, we assume with Fine (2012) that the mediate, partial explanatory-connective is definable on the basis of the full and immediate explanatory-connective.

[38]Notice that on this view we cannot hold either, that $T\ulcorner\Phi\urcorner \wedge T\ulcorner\Psi\urcorner$ explains $T\ulcorner\Phi \wedge \Psi\urcorner$ or, alternatively, that these two statements are explanatorily equivalent. It seems to us that any viable account of truth in non-causal explanations should be compatible with one of the two views.

[39]Incidentally, the typed framework we are working in facilitates drawing the distinction between horizontal and vertical explanations: let a proposition of level 0 be a proposition in which the truth predicate does not occur and a proposition of level $n + 1$ be a proposition where the truth predicate is applied to a proposition of level n. Horizontal explanations between propositions of level n can be expressed using the *is grounded in*-predicate of level $n + 1$. Vertical explanations between propositions of level n and propositions of level $n + 1$ cannot. They need to be expressed by an *is grounding in*-predicate of level $n + 2$.

We can maintain that according to Aristotelian Truth semantic ascent is explanatorily directed but this does not undermine our central theoretical tenet that it does not matter whether we perform semantic ascent globally or locally.

4.2 Substitution Axioms

So far we have introduced the basic logic characterizing the because connective and presented the truth theory $\mathsf{CT}_{<}$. Unfortunately, without further assumptions the logic and theory provide us with an unsatisfactory picture of the interaction of the because connective and the truth-predicate. For example, we cannot prove that the axioms of the logic of 'because' are true. As we have mentioned at the beginning of Section 4, this undermines one of the essential aspects of the deflationary truth predicate, which is to express infinite conjunctions and universal generalizations. The problem is mostly due to the fact that the logic of 'because' licenses no substitutions of equivalent—in whatever sense—sentences in the scope of the because connective and, more precisely, that no truth-theoretic statements $\mathsf{CT}_{<}$ deems equivalent can be substituted *salva veritate* in the scope of the because connective. To obtain a more satisfactory picture of the interaction of the truth predicate and the because connective we need to stipulate which truth-theoretic transformations are admissible from the perspective of the because connective or, in more philosophical terms, we need to specify which truth-theoretic statements are explanatorily equivalent. As we preempted at the end of Section 3 this leads us to a number of *Substitution axioms*.

But which truth-theoretic statements are equivalent from the explanatory perspective? At this point we propose to appeal again to our theoretical justification of the axiom ($\mathrm{T}_{<}$) discussed in Section 4.1 : we proposed that from the explanatory perspective there was no difference between performing semantic ascent locally or globally. But then, pushing this idea one step further, there should be no difference between the sentence $\mathrm{T}\ulcorner\neg\Phi\urcorner$ and the sentence $\neg\mathrm{T}\ulcorner\Phi\urcorner$. Whether we first semantically ascent from Φ to $\mathrm{T}\ulcorner\Phi\urcorner$ and then negate or whether we start with the negated sentence $\neg\Phi$ and then semantically ascent should make no difference from the explanatory perspective. This kind of argument can be extended to all logical connectives and also the quantifiers. Formally, this can be captured by four admittedly inelegant principles that assert that the logical connectives and quantifiers can be moved in and out of the scope of the truth predicate within the scope of the because connective. For ease of presentation we introduce a three-place logical operator Sub such that $\mathsf{Sub}(A, B, C)$ denotes the formula that results from substituting the formula C for the formula B in the formula A, i.e.,

$$\mathsf{Sub}(A, B, C) := A(C/B).$$[40]

The so-called substitution axioms can then be stated as follows:

($\mathrm{Sub}_{\neg}$) $\quad \forall\varphi\,(\,\mathsf{Sub}(A < B, C, \mathrm{T}[\neg\varphi]) \leftrightarrow \mathsf{Sub}(A < B, C, \neg\mathrm{T}[\varphi])\,)$

($\mathrm{Sub}_{\wedge}$) $\quad \forall\varphi,\psi\,(\,\mathsf{Sub}(A < B, C, \mathrm{T}[\varphi\wedge\psi]) \leftrightarrow \mathsf{Sub}(A < B, C, \mathrm{T}[\varphi]\wedge\mathrm{T}[\psi])\,)$

($\mathrm{Sub}_{<}$) $\quad \forall\varphi,\psi\,(\,\mathsf{Sub}(A < B, C, \mathrm{T}[\varphi < \psi]) \leftrightarrow \mathsf{Sub}(A < B, C, \mathrm{T}[\varphi] < \mathrm{T}[\psi])\,)$

[40] A, B, C are formulas and may thus contain free variables. $\mathsf{Sub}(A, B, C(\vec{x}))$ denotes the substitution of $C(\vec{x})$ for B in A where x is new to A.

$$(\mathrm{Sub}_{\forall}) \qquad \forall\varphi(v)\,(\,\mathsf{Sub}(A \prec B, C, \mathrm{T}[\forall v\varphi(v)]) \leftrightarrow \mathsf{Sub}(A \prec B, C, \forall y\mathrm{T}[\varphi(\dot{y}/v)]),)\,.$$

To illustrate these substitution principles let us look at an example and assume with Horwich that

(18) ⌜Snow is white and grass is green⌝ is true because snow is white and grass is green.

Formally, this can be rendered as

$$(*) \qquad \Phi \wedge \Psi \prec \mathrm{T}\ulcorner\Phi \wedge \Psi\urcorner$$

where Φ stands for ‘snow is white’ and Ψ for ‘grass is green’. Now, the Substitution axioms in the form of ($\mathrm{Sub}_{\wedge}$) allow us to infer that (∗) is equivalent to

$$(**) \qquad \Phi \wedge \Psi \prec \mathrm{T}\ulcorner\Phi\urcorner \wedge \mathrm{T}\ulcorner\Psi\urcorner,$$

that is, the explanation

(19) ⌜Snow is white⌝ is true and ⌜grass is green⌝ is true because snow is white and grass is green.

The Substitution axioms make the logical structure of a sentence in the scope of the truth predicate *transparent* from the perspective of the because connective. They do not render the truth predicate itself transparent, i.e., Φ and $\mathrm{T}\ulcorner\Phi\urcorner$ are not generally substitutable in the scope of the because connective. If this were the case, we could no longer square our proposal with Horwich’s view we just appealed to, namely, that Φ explains $\mathrm{T}\ulcorner\Phi\urcorner$. Jointly the two assumptions would be in conflict with the irreflexivity of the because connective. Ultimately, these two conflicting ideas will lead to the two different accounts of the explanatory role of the truth predicate in non-causal explanations we have already mentioned: *Strongly Transparent Truth* and *Aristotelian Truth.*

However, if these four axioms are added to the logic of ‘because’ we can prove true the basic axioms of the logic of ‘because’ in the theory $\mathsf{CT}_{\prec}$ and, at least from this perspective, $\mathsf{CT}_{\prec}$ now seems to be an acceptable truth theory for the language of the because connective. For example, in this framework the theory proves the axiom (≺¬) true:

$$\forall\varphi\mathrm{T}[\varphi \longrightarrow (\varphi \prec \neg\neg\varphi)].$$

Of course, since we are working with a typed truth predicate it will not be possible to prove true the Substitution axioms themselves. But as far as the language without the truth predicate is concerned the theory proves all axioms of the logic true and this is all that can be expected in the typed setting. Moreover, the set up is also sufficient for showing that the *is grounded in*-predicate defined along the lines of (DefG) has the common logical properties of a partial grounding relation discussed in the literature. Unfortunately though, our truth predicate is still unsatisfactory from a deflationary perspective.

4.3 Reaching Deflationary Truth

The T-scheme is the constitutive principle of deflationary truth and is often thought to fully capture the deflationary notion of truth. But we argued that the deflationist is also in need of compositional principles of truth to account for the full expressive function of the truth predicate, and it was for this reason that we based our theory of truth on the compositional principles rather than the T-scheme. In extensional languages this is an acceptable maneuver since in this context the T-scheme will be a consequence of the compositional principles. But in the context of the hyperintensional language of the because connective the situation changes and even assuming the Substitution axioms we can no longer derive the T-scheme within the compositional theory $\mathsf{CT}_{<}$. As a consequence, the theory is unsatisfactory from the deflationary perspective.

A straightforward fix would be to simply add the T-scheme as an additional axiom and perhaps, from a philosophical perspective, this does not constitute a substantial problem. After all, according to the deflationist the T-scheme is the most basic feature of the truth predicate and the fact that it is not a consequence of the compositional axioms once we consider the because connective is not a threat to their doctrine. Indeed, if we adopt this point of view it suffices to adopt a version of the T-scheme as an additional axiom, which is restricted to sentences of the form $\Phi < \Psi$: for all sentences Φ, Ψ of the language without the truth predicate

$$(\mathsf{TS}^*) \qquad \mathrm{T}\ulcorner \Phi < \Psi \urcorner \leftrightarrow (\Phi < \Psi).$$

The resulting theory, which we call $\mathsf{CT}^*_{<}$, should be deflationary acceptable and also proves sufficient for defining a partial grounding-predicate that meets our requirements.

While adding (TS^*) to the theory $\mathsf{CT}_{<}$ yields a viable strategy for reaching deflationary truth one may disagree with the underlying diagnosis and hope for a more elegant way of recovering the T-scheme. Rather than blaming the truth theory for the fact that the T-scheme does not follow from the compositional axioms one could equally hold the logic responsible: according to this view the Substitution axioms introduced in the previous section are not sufficient for aptly characterizing the explanatory role of the truth predicate in non-causal explanations, that is, we have to introduce further axioms to the logic of 'because'. This leads to the fork between the two mutually incompatible accounts of truth in non-causal explanations we already anticipated: *Strongly Transparent Truth* and *Aristotelian Truth.*

4.3.1 Strongly Transparent Truth

So far we have championed the view that from an explanatory perspective there is no difference between performing semantic ascent locally or globally and we argued that this renders the logical structure of a sentence in the scope of the truth predicate transparent to the because connective: as far as logical structure is concerned the truth predicate is invisible in non-causal explanations. Strongly Transparent Truth takes this idea of transparency even further and holds that the truth predicate is fully explanatorily transparent. It is invisible to the because connective. As a consequence, Φ and $\mathrm{T}\ulcorner\Phi\urcorner$ are explanatorily equivalent for every sentence Φ and can be substituted *salva veritate* in the scope of the because connective. Technically this can be achieved by introducing a further Substitution axiom

$$(\text{Sub}_{\mathsf{At}}) \qquad \forall x, y\,(\,\mathsf{Sub}(A \prec B, \Psi, x = y) \leftrightarrow \mathsf{Sub}(A \prec B, \Psi, \mathrm{T}[\dot{x} = \dot{y}])\,)\,.$$

(Sub_{At}) together with the other Substitution axioms yields that Φ and $\mathrm{T}\ulcorner\Phi\urcorner$ can be substituted *salva veritate* in the scope of the because connective, that is, we can prove:

$$(\text{Sub}_{\mathsf{TS}}) \qquad \mathsf{Sub}(A \prec B, \Psi, \Phi) \leftrightarrow \mathsf{Sub}(A \prec B, \Psi, \mathrm{T}\ulcorner\Phi\urcorner).$$

If the logic of 'because' is supplemented by the principle (Sub_{At}) in combination with the Substitution axioms for the logical connectives and quantifiers, the theory $\mathsf{CT}_{\prec}$ proves sufficient for deriving the T-scheme for all sentences Φ of the language without the truth predicate.

Clearly, Strongly Transparent Truth takes the deflationist's idea that truth is merely an expressive device to its extreme and, presumably, will not appeal to all deflationists. But deflationists that are willing to deem the intuitions in support of correspondence-theoretic truth to be not only metaphysically but also linguistically confused might be tempted by the view: the truth predicate simply has no explanatory role be it in causal or non-causal explanations. There is just one role of the truth predicate and it is to equip the language with essential expressive resources. While this is an extreme view it provides us with a coherent theoretical picture of how the T-scheme follows from the theory $\mathsf{CT}_{\prec}$ once the explanatory role of the truth predicate is sufficiently specified.

4.3.2 Aristotelian Truth

Strongly Transparent Truth clashes with perhaps the most basic linguistic intuition in favor of correspondence truth and rather than attempting to dissolve this conflict it declares the intuition to be outright confused. The basic intuition at stake is that it is true that Φ because Φ, e.g., it is true that snow is white because snow is white. The principle is sometimes called the principle of Aristotelian Truth (Künne, 2003; Schnieder, 2011) and can formally be spelled out as follows:

$$(\prec\mathrm{T}\!\downarrow) \qquad \mathrm{T}\ulcorner A\urcorner \rightarrow (A \prec \mathrm{T}\ulcorner A\urcorner).$$ [41]

As we have mentioned earlier in this paper, Aristotelian Truth and the conception of Strongly Transparent Truth taken together would violate the irreflexivity of the because connective, that is, assuming ($\prec\mathrm{T}\!\downarrow$) and ($\text{Sub}_{\mathsf{TS}}$) $A \prec A$ can be derived for every true sentence A. So if, following Horwich (1998b, 2010), one acknowledges that there is some initial plausibility to the linguistic correspondence-intuition and thus subscribes to the principle of Aristotelian Truth, the view of Strongly Transparent Truth must be resisted. In this case we have two options to proceed. The first option would be to accept that the T-scheme cannot be derived in the theory $\mathsf{CT}_{\prec}$ and to adopt the theory $\mathsf{CT}^{*}_{\prec}$. The second option is to make the T-scheme depend solely on the logic of 'because'. In this case we need to introduce a further axiom to the logic of 'because', namely,

$$(\prec\mathrm{T}\!\uparrow) \qquad \Phi \rightarrow (\Phi \prec \mathrm{T}\ulcorner\Phi\urcorner).$$ [42]

[41] Notice that ($\prec\mathrm{T}\!\downarrow$), in contrast to ($\prec\mathrm{T}\!\uparrow$) below, is formulated for all formulas of the language with the truth predicate. This is acceptable since ($\prec\mathrm{T}\!\downarrow$) will be trivially true if the truth predicate occurs in A. This follows from axiom (Snt). Cf. Footnote 31.

($\prec$T$\uparrow$) is simply an alternative formulation of the principle of Aristotelian Truth, which omits explicit mention of the truth predicate in the antecedent condition. But jointly the two principles of Aristotelian Truth together with the factivity of the because connective imply the T-scheme for all sentences of the language without the truth predicate.[43] On this account, the T-scheme would be completely independent of the theory of truth we adopt and, in some sense, a purely logical principle. While perhaps this sounds appealing to some deflationists who hold truth to be a logical notion, we think this account should be resisted—at least if one wishes to follow Horwich (1998b): according to Horwich it is a consequence of his minimal theory that principles ($\prec$T$\downarrow$) and ($\prec$T$\uparrow$) are true. The T-scheme should not be a consequence of the logic of 'because' but should depend, at least to some extent, on our theoretical assumptions regarding the notion of truth, that is, our theory of truth. In conclusion, it seems that the proponent of Aristotelian Truth should adopt the theory $\mathsf{CT}^*_\prec$ as their theory of truth. This might be unsatisfactory from a formal perspective but we see no problem with this strategy from the more philosophical perspective.

In our discussion of the explanatory role of the truth predicate we have only touched upon a wealth of issues concerning the interaction of a deflationary truth predicate and a non-causal because connective but we hope to have provided some foundations for fruitful future investigation. Moreover, the framework we developed proves sufficient for defining a deflationary *is grounded in*-predicate. Before we elaborate on this remark we point out that, as we show in Appendix A, the formal framework does indeed yield coherent and consistent theories of truth for a language of a hyperintensional because connective.

5 Ground

In the previous sections we have set the stage for defining and evaluating a deflationary *is grounded in*-predicate along the lines of the informal definition (DefG) put forward in the Introduction. In this section we argue that the deflationary *is grounded in*-predicate is up for the task—at least from a logical perspective. It has the same logical properties, i.e., plays the same logical role as the ground-predicate that grounding-theorists assume to aptly express the metaphysical relation of partial grounding. The definition

(D$\lhd$) $$x \lhd y \;:\leftrightarrow \mathrm{T}x \prec \mathrm{T}y,$$

is the exact formal counterpart of (DefG) where 'is grounded in' is the intended reading of '$\lhd$'. Now, we can indeed show that the logical laws for partial ground commonly assumed in the literature can be proved for the deflationary *is grounded in*-predicate '$\lhd$' in the theory $\mathsf{CT}^*_\prec$ on the basis of the logic of 'because' extended by the Substitution axioms for the logical connectives and quantifiers. The choice of $\mathsf{CT}^*_\prec$ guarantees that our observation applies to both Aristotelean and Strongly Transparent Truth. This may come as a surprise, since the two views seem to have, as discussed, very different accounts of the explanatory role of the truth predicate in non-causal explanations. However, this difference will only show if we try to define an *is*

[42]Recall that $A, B, C, \dots$ are schematic variables for formulas of the language with the truth predicate, while $\Phi, \Psi, \dots$ stand for sentences of the language without the truth predicate.

[43]($\prec$T$\downarrow$), due to (FactL), implies $\mathrm{T}\ulcorner\Phi\urcorner \rightarrow \Phi$, while ($\prec$T$\uparrow$) together with (FactL) implies $\Phi \rightarrow \mathrm{T}\ulcorner\Phi\urcorner$.

grounded in-predicate for the language including the truth predicate whilst in this paper our aim is to define an *is grounded in*-predicate for the language without the truth predicate.

The properties of partial ground assumed in the literature are closely related to those of the because connective we have discussed in Section 2 (Fine, 2012; Korbmacher, 2017). For example, the partial ground-predicate is also supposed to be irreflexive, transitive and factive and we can show that the $\lhd$-predicate has all these properties. But in contrast to the case of the because-operator these properties can now be stated in universally quantified form:

(IR$_\lhd$) $\forall\varphi(\neg(\varphi \lhd \varphi))$,

(Trans$_\lhd$) $\forall\varphi,\psi,\chi((\varphi \lhd \psi) \rightarrow ((\psi \lhd \chi) \rightarrow (\varphi \lhd \chi)))$,

(FactL$_\lhd$) $\forall\varphi,\psi((\varphi \lhd \psi) \rightarrow \mathrm{T}\varphi)$,

(FactR$_\lhd$) $\forall\varphi,\psi((\varphi \lhd \psi) \rightarrow \mathrm{T}\psi)$.

More generally, all the laws of partial ground assumed in the most explicit extant formal account of the partial ground-predicate due to Korbmacher (2017) can be proved for the $\lhd$-predicate in the theory $\mathsf{CT}^*_\lhd$.[44] We take this to show that from a logical perspective the $\lhd$-predicate is an adequate partial ground-predicate. The deflationary notion of ground is, at least when it comes to the logical role, up to the job the metaphysical notion of ground is meant to play.[45]

However, the proponent of metaphysical grounding may resist this conclusion on the ground that while we have provided some evidence that the work of the partial notion of ground can be accounted for in deflationary terms in this paper, the more important notion of ground is that of *full and immediate ground* (cf. Section 4.1.1). It is precisely when we turn to full and immediate ground, the grounding-theorist will argue, that we need to appeal to non-deflationary and more substantial assumptions.

[44] More precisely, Korbmacher's theory PG can be interpreted in $\mathsf{CT}^*_\lhd$. This also yields an alternative and to our minds simpler consistency proof for the theory PG.

[45] Grounding-theorists (see, e.g., Correia, 2017) typically supplement their account of grounding by a notion of ground-theoretic equivalence, that is, they specify conditions for when formulas can be substituted *salva veritate* within explanatory contexts. For example, they might hold that $\Phi\wedge\Psi$ and $\Psi\wedge\Phi$ are ground-theoretically equivalent. A notion of ground-theoretic equivalence can be introduced to our setting by either introducing further substitution axioms or, more elegantly, by directly defining such a relation on the bearers of truth. For example, we could define ground-theoretic equivalence $\approx$ along the line of Poggiolesi (2016, 2018, 2022) as follows:

(D$_\approx$) $\ulcorner\Phi\urcorner \approx \ulcorner\Psi\urcorner :\leftrightarrow (\mathrm{T}\ulcorner\Phi\urcorner \leftrightarrow \mathrm{T}\ulcorner\Psi\urcorner) \wedge g\text{-}\mathsf{Com}(\ulcorner\Phi\urcorner) = g\text{-}\mathsf{Com}(\ulcorner\Psi\urcorner)$.[46]

and the supplement the theories $\mathsf{CT}_\lhd$ and $\mathsf{CT}^*_\lhd$ by the first-order substitution principle

($\mathsf{Subst}_\approx$) $\forall x, y(x \approx y \rightarrow (A(x) \rightarrow A(y)))$.

The consistency of the resulting theories can be shown by following the outlines of the consistency proof in the Appendix, but where the underlying notion of complexity is replaced by Poggiolesi's notion of grounding complexity. Which notion of ground-theoretical equivalence is adopted will arguable depend on the particular explanatory context, but such a notion will not pose a particular challenge to our proposal.

5.1 Full Ground and Immediate Ground

The gist of our proposal was to define an *is grounded in*-predicate by nominalizing the argument positions of the because connective as it is used in non-causal explanations. However, it is clear that this strategy will not yield a predicate of full ground, let alone of full and immediate ground. 'because' in natural language has but two argument positions and, as a consequence, the ground-predicate we obtain by nominalizing the argument positions will have two argument positions likewise. In contrast the full ground of, say, a conjunction will usually consist in both its conjuncts. Similarly, the standard view on full ground has it that a general proposition will be fully grounded jointly by all its instances. This means that the explanans can potentially consist of an infinity of propositions and, clearly, there will be no explanatory operator in natural language that accommodates this theoretical need (nor will there be such an operator in some formal finitary first-order language). Does this mean that a deflationary predicate of full ground is out of reach and that we need substantial metaphysical assumptions to move beyond a partial ground-predicate?

As we shall see, an answer to this question will depend on which abstraction and idealization processes one deems theoretically acceptable. We started by considering the uses of 'because' in non-causal explanations and as a matter of fact these explanations will, more often than not, be partial in character. We thus ended up with a partial notion of ground. Yet, at least from a formal perspective we take it to be a fairly standard abstraction and generalization process to, starting from the properties of the because-operator, extrapolate the laws and properties an explanatory operator would have, if we were working in idealized circumstances in which we are concerned with non-partial, i.e., full explanations. The idea would be that the transition from a partial to a full explanatory connective is comparable to the transition from finitary proof systems to infinitary ones in mathematical logic, which is very well understood.[47] Of course, the notion of proof in such infinitary systems is no longer decidable, but this does not imply that these systems are obscure or esoteric in any relevant sense. However, one may still worry that while abstracting from the properties of a finitary notion (partial ground) to an infinitary notion (full ground) in a controlled environment such as proof systems of first-order logic is unproblematic, such abstraction and idealizations should not be assumed to be unproblematic in non-causal explanations more generally. Evaluating the charge would require an in-depth study of scientific methodology and non-causal explanations. This goes beyond the scope of our paper but suffices it to say that this might be where the points of disagreement between grounding-theorist and grounding-skeptic become apparent.

Putting this discussion aside, if we were to avail ourselves to a *generalized because-operator*, i.e., an explanatory connective for full, non-causal explanations, call it $<_\infty$, a predicate of full ground could be defined along the lines of definition (D$\lhd$).[48] The resulting grounding predicate will be a multigrade predicate and allow for a variety of different, possibly infinite, arities as

[47] See, for instance, Troelstra and Schwichtenberg (2000) for a discussion of the connection between finitary and infinitary proof systems.

[48] If Schnieder's (2011) logic of 'because' is accepted as an apt characterization of the because-operator, the generalized because-operator will presumably be similar to Fine's (2012) grounding-operator.

would the generalized because-operator:

$$(\mathsf{D}\triangleleft_\infty) \qquad x_1, x_2, \cdots \triangleleft_\infty y \;:\leftrightarrow\; \mathrm{T}x_1, \mathrm{T}x_2, \ldots <_\infty \mathrm{T}y.$$

As we have just argued, it seems rather unreasonable to hold that the abstraction and generalization process leading to a generalized because-operator forces us to indulge in esoteric or substantial metaphysics. But then the full ground-predicate of ($\mathsf{D}\triangleleft_\infty$) seems deflationary acceptable and thus considering the notion of full ground should not cause any particular problems for a deflationary view of ground.

However, while a definition of a full ground predicate along the lines of ($\mathsf{D}\triangleleft_\infty$) may not force esoteric or substantial metaphysical assumption upon us it may undermine one of the basic motivations for truth-theoretic deflationism. According to most truth-theoretic deflationists the raison d'être of the truth predicate is to express infinite conjunctions and disjunctions we could not express otherwise in our language. But the definition ($\mathsf{D}\triangleleft_\infty$) requires an infinitary language and in such a language it is usually possible to formulate infinite conjunctions and disjunctions. But then, it seems, the deflationary truth predicate is redundant and should be omitted.

In response to this objection it is worth noting that it is not an argument against a deflationary view of ground *per se* but against a particular way of conceiving of the ground-predicate. Although, admittedly, since we have based our case for a deflationary perspective of ground on definition ($\mathsf{D}\triangleleft_\infty$), a positive account of the deflationary ground-predicate is left wanting if the objection is granted. To evaluate the objection it is helpful to reconsider the reasons why a full ground-predicate cannot be defined on the basis of the because connective or any other explanatory connective based on expressions employed in natural language. So far we have mainly blamed the number of argument positions of the because connective for this failure, yet the issue is slightly more subtle. As Fine puts it *" 'because'... is not able to distinguish between a single conjunctive antecedent and a plurality of non-conjunctive antecedents."* (Fine, 2012, p. 46) Fine's point is that in natural language there seems to be no obvious way of distinguishing between a conjunction as opposed to a list of individual arguments of an operator: we convey the list by conjoining the individual arguments by the word 'and'. But we also use the word 'and' if the individual arguments of the list were to form one single conjunction. To avoid any confusion let us call the former, improper conjunction a *metalinguistic conjunction* and the latter, ordinary conjunction an *object-linguistic conjunction*. Now, the full notion of ground requires the metalinguistic conjunction as opposed to the object-linguistic notion and, as Fine points out, we cannot convey the metalinguistic conjunction of arguments without conflating it to the common object-linguistic conjunction. The difference between the object- and metalinguistic conjunction also affects the infinitary case, that is, in this case we may also distinguish between a conjunction with an infinite number of conjuncts and an infinite list of propositions or arguments.[49] On our view, the expressive function of the truth predicate is to formulate infinite *object-linguistic* conjunctions and disjunctions as opposed to metalinguistic ones. So before we answer the objection against our proposal it is important to note that the failure of defining a full ground-predicate on the basis of the because connective does not highlight

[49]Basically, this point is already made in a slightly different context by Gupta (1993a) who uses this distinction to argue against deflationary conceptions of truth.

a problem with truth-theoretic deflationism as such: the deflationary truth predicate is not a tool for expressing infinite metalinguistic conjunctions and we should therefore not expect a full ground-predicate to be definable on the basis of the deflationary truth predicate and the because connective of natural language.[50]

With this in mind let us return to the objection that the definition ($\mathsf{D}_{\triangleleft_\infty}$) undermines the need of a deflationary truth predicate in the language and that, as a consequence, the definition is self-undermining: the definition does not yield a deflationary full ground-predicate. But the objection puts the cart before the horse because the deflationist in no way needs to accept the claim that their language is expressively complete and, in particular, that metalinguistic conjunctions ought to be expressible in their language. The only claim that our deflationist explicitly endorses is that in their language the function of the truth predicate is to express infinite object-linguistic conjunctions and disjunctions. It may well be that for theoretical purposes we need to extend the language and perhaps introduce infinitary languages but this does not affect the intelligibility of the deflationary truth predicate or its rationale. In other words, even within an infinitary language we may still have a deflationary truth predicate and definition ($\mathsf{D}_{\triangleleft_\infty}$) should not be disqualified on the basis of the legitimacy of the deflationary truth predicate. Summing up it seems to us that in virtue of ($\mathsf{D}_{\triangleleft_\infty}$) deflationists are licensed to the notion of full ground.

What about immediate ground? According to the grounding-theorist (Correia, 2010; Fine, 2012) a fact $\ulcorner\Phi\urcorner$ immediately grounds the fact $\ulcorner\Psi\urcorner$, if the grounding of the fact $\ulcorner\Psi\urcorner$ by the fact $\ulcorner\Phi\urcorner$ is not mediated via some other fact.[51] We related full ground to the generalized because-operator, which, we argued, could be obtained via a fairly standard abstraction process from the because-operator figuring in non-causal explanations. Can the same be done for immediate ground, i.e., can we obtain an immediate because-operator by a similar kind of abstraction process that led us to the generalized because-operator?

If we return to the analogy of proof systems for first-order logic, then one may be tempted to give a positive answer: while a mediate ground corresponds to some previous node in the proof tree of the explanandum, an immediate ground is simply a node that is an immediate predecessor in the proof tree. Incidentally, this idea seems to be driving Fine's account of

[50]That said, it would be possible to conceive of the truth predicate as a device for expressing metalinguistic rather than object-linguistic conjunctions. But this view clashes with our account of the explanatory role of the truth predicate in non-causal explanations for we would need to give the axioms of $\mathsf{CT}_{\prec}$ an explanatory reading, e.g., we would have

$$\forall\varphi,\psi(\mathrm{T}[\varphi]\wedge\mathrm{T}[\psi]\prec\mathrm{T}[\varphi\wedge\psi])$$
$$\forall\varphi(v)(\forall y\mathrm{T}[\varphi(\dot{y}/v)]\prec\mathrm{T}[\forall v\varphi]).$$

This view would potentially allow for a predicate of full ground without resorting to an infinitary language. But, at least prima facie, there seems to be some tension between truth-theoretic deflationism and giving the compositional principles an explanatory, i.e. truth-conditional, reading. Although perhaps this conflict is only superficial since we are considering non-causal rather than causal explanations.

[51]The notion of immediate ground, in contrast to the notion of mediate, partial ground we discussed in the paper, will not be transitive. The notion of mediate ground can be defined as the transitive closure of the notion of immediate ground. Notice that a ground $\ulcorner\Phi\urcorner$ can be an immediate and a mediate ground at the same time: $\ulcorner\Phi\urcorner$ immediately grounds $\ulcorner\Phi\wedge(\Phi\wedge\Psi)\urcorner$ but $\ulcorner\Phi\urcorner$ is also a ground of $\ulcorner\Phi\wedge\Psi\urcorner$ and hence mediately grounds $\ulcorner\Phi\wedge(\Phi\wedge\Psi)\urcorner$ via $\ulcorner\Phi\wedge\Psi\urcorner$ (cf. Fine, 2012).

immediate ground. But one might again be worried to what extent this simple picture provides us with a good model of non-causal explanations and explanatory structures more generally.[52] We feel that in the case of the notion of immediate ground the worry is somewhat more pressing than in the case of full ground: it is unclear whether such immediate explanations can play an important theoretical role once we move away from the usual toy examples. The worry is that once we move away from toy examples towards more complicated and realistic cases of explanation there may be no clear cases of immediate explanation, that is, of immediate ground. As a consequence, immediate ground cannot be assigned the foundational role grounding-theorists usually attribute to it.[53] Some further support for this assessment stems from the fact that in Section 4.1.1 we distinguished between horizontal and vertical immediate explanations, suggesting that the notion of immediate explanation is not as simple as some of the usual examples suggest.

However, this more skeptical stance toward the notion of immediate ground does not undermine the proposed reconstruction of the metaphysicians ground-predicate in terms of an explanatory because connective and the truth predicate. If we deem an immediate because-operator to be a useful and a well-motivated tool for analyzing non-causal explanations, then a deflationary immediate ground-predicate can be obtained along the lines of the strategy entertained in this paper. In conclusion, while a more skeptical stance towards the notion of immediate ground does not lead to a technical hurdle in defining a predicate of immediate ground along the lines of (DefG), it highlights a point where there is bound to be substantial disagreement between grounding-theorist and grounding-skeptic: grounding-skeptic and grounding-theorist will disagree on whether an "immediate-because connective" can be obtained on the basis of the explanatory connective characterizing the use of 'because' in non-casual explanations via reasonable abstraction and idealization processes. We are left with the genuine philosophical question of whether a convincing case in support of the notion of immediate ground and to some lesser extent the notion of full ground can be made. The way we have cast the debate an answer to this question will depend on which abstraction and idealization processes one deems acceptable in moving to accounts of explanations that are no longer directly based on our use of 'because' in non-causal explanations. Perhaps focussing on this latter question will allow for a more constructive debate grounding-theorist and grounding-skeptic.

6 Conclusion

In the paper we proposed understanding the metaphysician's ground-predicate in terms of the because connective of non-causal explanations and the truth predicate. We assumed a form of methodological deflationism with respect to the notion of truth and consequently, so we

[52]Incidentally, this kind of worry seems to be at the heart of Hofweber's (2009) criticism of Fine's "esoteric metaphysics".

[53]That is, we don't share Fine's assessment for whom it is remarkable *"how strong our intuitions are about when it* [the notion of immediate ground, JS] *does and does not hold."* (Fine, 2012) For example, at least prima facie it seems that in order to make sense of the notion of immediate ground that we need to presuppose "explanatory structures" such that every set of elements of the structure has a least upper bound. But need that always be the case and, more to the point, should that not be the outcome of an investigation of grounding and explanatory structures rather than presupposed at the outset of the investigation?

argued, the notion of ground. On this proposal the distinct metaphysical sense the grounding-theorist associates with the notion of ground would be tied to the correspondence intuition associated with the truth predicate—an intuition a truth-theoretic deflationist would qualify as misleading (and ill-founded). We remained and remain neutral on whether the deflationary account of ground (and of truth) is successful, but we hope to have shown in this paper that ground-theoretic methodological deflationism will not falter because of logical reasons: from a logical perspective the deflationary *is grounded in*-predicate is up to the job.[54]

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[54]In this paper we have only considered non-iterated grounding and, ultimately, a deflationary account of ground should develop a theory of iterated ground. Admittedly, this is no simple affair because of the self-referential paradoxes. It is, however, a straightforward matter to introduce a ground-hierarchy, i.e., a hierarchy of different *is grounded in*-predicates corresponding to a hierarchy of truth predicates. Perhaps this is not the most satisfactory approach, but the distinction between horizontal and vertical explanations we appealed to in Section 4.1.1 suggests that we need to have some bookkeeping of semantic levels in place. This may provide independent support for the typing approach.
If we try to define an *is grounded in*-predicate in a theory of self-applicable truth problems arise since the truth predicate may no longer commute with negation. To construct a theory of truth for the language of the because connective and to define a reasonable *is grounded in*-predicate then requires a dual notion for the because connective, i.e., a notion that relates to 'because' in the way 'possibility' relates to 'necessity'—see Stern (2016, 2014) for a discussion of these issues with respect to modal notions. Unfortunately, it is unclear what this dual notion could be. Perhaps it is therefore more promising to abandon classical logic and move to some theory of truth formulated in non-classical, arguably substructural, logic but we leave the discussion of iterated ground for another occasion. See, e.g., Litland (2015, 2017) for some discussion of the logic and philosophy of iterated grounding.

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A Models and Consistency

In this section we show the consistency of our theory of truth in the logic of 'because', but first we introduce some notation. In this section $\mathcal{L}$ is assumed to be the arithmetical language extended by the $<$-connective, $\mathcal{L}_T$ is $\mathcal{L}$ extended by the truth predicate. The language is assumed to have function symbols representing some basic syntactic operation. We assume some standard coding scheme that assigns every expression η of the language a natural number $\#\eta$. $\#\eta$ is called the code of η, that is, its Gödel number. $\ulcorner\eta\urcorner$ is the numeral of the Gödel number of η. For further notational conventions we refer back to Footnotes 28 and 31, and, again, Halbach (2014) for some general background. We denote the basic logic of 'because', that is, the logic without the Substitution axioms by BC; BC together with *all* Substitution axioms will be called BCT; BC together with the Substitution axioms for the logical connectives and quantifiers and the principle ($<T\downarrow$) but *without* ($\mathrm{Sub}_{\mathrm{At}}$) will be called BCA. $\mathcal{N}$ denotes the standard model of arithmetic.

Proposition A.1. *There exists* $\mathsf{Tr} \subseteq \omega$ *such that for* $A \in \mathcal{L}_T$

(*i*) $$\mathsf{CT}_< \vdash_{\mathsf{BCT}} A \Rightarrow (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{T}} A;$$

(*ii*) $$\mathsf{CT}^*_< \vdash_{\mathsf{BCA}} A \Rightarrow (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} A.$$

$\models_{\mathsf{T}}$ ($\models_{\mathsf{A}}$) denotes the relation of truth in a model according to the conception of Strongly Transparent Truth (Aristotelian Truth). The set Tr will simply be the Tarskian truth set, i.e., the set of true sentences of $\mathcal{L}$. To define this set we need to introduce the truth-conditions for the $<$-connective. The interpretation of $<$ modifies a trick used by Schnieder (2011) to prove the consistency of BC and is not to be understood as the intended interpretation of the $<$-connective. In contrast to Schnieder (2011) we also have deal with sentences of $\mathcal{L}_{\mathrm{T}}$ since the logic of 'because' is formulated for the entire language. We thus need to assign a complexity to sentences in which the truth predicate occurs. This will be done by translating every sentence of $\mathcal{L}_{\mathrm{T}}$ into a sentence of $\mathcal{L}$, i.e., by the translation function $\mu : \{\mathrm{T}t : t \in \mathsf{ClosedTerm}_{\mathcal{L}_{\mathrm{T}}}\} \longrightarrow \mathsf{Sent}_{\mathcal{L}}$ with

$$\mu(\mathrm{T}t) := \begin{cases} \Phi & \text{if } t^{\mathcal{N}} \in \mathsf{Sent}_{\mathcal{L}} \;\&\; t^{\mathcal{N}} = \#\Phi \\ 0 = 1 & \text{otherwise} \end{cases}$$

So μ either simply disquotes the sentence in scope of the truth predicate, if the latter is a sentence of the language $\mathcal{L}$ or assigns a (false) atomic sentence to $\mathrm{T}t$. The idea of the latter translation is that the truth predicate is typed and hence if a term t does not denote a sentence of $\mathcal{L}$, we treat it as if it were not a sentence at all. In this case $\mathrm{T}t$ is just a simple atomic sentence of the language.

We now define the complexity of a sentence of $\mathcal{L}_{\mathrm{T}}$.

Definition A.2 (Complexity)**.** *We introduce two different notions of complexity* $\mathsf{Com}_{\mathsf{T}}$ *and* $\mathsf{Com}_{\mathsf{A}}$ *for* BCT *and* BCA *respectively.* $\mathsf{Com}_{\mathsf{T}}$ *and* $\mathsf{Com}_{\mathsf{A}}$ *are functions that assign to each sentence of* $\mathcal{L}_{\mathrm{T}}$ *some ordinal number.*

$$\mathsf{Com}_{\mathsf{T}}(A) := \begin{cases} 0, & \textit{if } A \doteq (s = t) \textit{ for } s, t \in \mathsf{ClosedTerm}_{\mathcal{L}_{\mathrm{T}}} \\ \mathsf{Com}_{\mathsf{T}}(B) + 1, & \textit{if } A \doteq \neg B \\ \mathsf{Max}(\{\mathsf{Com}_{\mathsf{T}}(B), \mathsf{Com}_{\mathsf{T}}(C)\}) + 1, & \textit{if } A \doteq B\mathbf{J}C, \mathbf{J} \in \{\wedge, <\} \\ \mathsf{Sup}(\{\mathsf{Com}_{\mathsf{T}}(B(t)) : t \in \mathsf{Closedterm}_{\mathcal{L}_{\mathrm{T}}}\}) + 1, & \textit{if } A \doteq \forall v B \\ \mathsf{Com}_{\mathsf{T}}(\mu(\mathrm{T}t)), & \textit{if } A \doteq \mathrm{T}t \textit{ for } t \in \mathsf{ClosedTerm}_{\mathcal{L}_{\mathrm{T}}}. \end{cases}$$

$$\mathsf{Com}_{\mathsf{A}}(A) := \begin{cases} 0, & \textit{if } \phi \doteq (s = t) \textit{ for } s, t \in \mathsf{ClosedTerm}_{\mathcal{L}_{\mathrm{T}}} \\ \mathsf{Com}_{\mathsf{A}}(B) + 1, & \textit{if } \phi \doteq \neg A \\ \mathsf{Max}(\{\mathsf{Com}_{\mathsf{A}}(B), \mathsf{Com}_{\mathsf{A}}(C)\}) + 1, & \textit{if } A \doteq B\mathbf{J}C, \mathbf{J} \in \{\wedge, <\} \\ \mathsf{Sup}(\{\mathsf{Com}_{\mathsf{A}}(B(t)) : t \in \mathsf{Cterm}_{\mathcal{L}_{\mathrm{T}}}\}) + 1, & \textit{if } A \doteq \forall v B \\ \mathsf{Com}_{\mathsf{A}}(\mu(\mathrm{T}t)) + 1, & \textit{if } A \doteq \mathrm{T}t \textit{ for } t \in \mathsf{ClosedTerm}_{\mathcal{L}_{\mathrm{T}}}. \end{cases}$$

The two notions of complexity will only diverge for sentences containing the truth predicate. Also notice that the complexity of the universal quantifier is defined in a non-standard way. Usually, it is defined as the maximum complexity of all its instance plus one. However, in the present case we are not guaranteed that there will be a maximum, for example in the case of the sentence $\forall x \mathrm{T}x$. By taking the complexity to be the supremum plus one, some universally quantified sentences will be assigned the complexity $\omega + 1$ and ultimately we continue counting from there. This means that the complexity of a sentence may grow up to, but not including, $\omega + \omega$. However it is worth noting that for sentences of the language $\mathcal{L}$ a maximum will always exists and hence our notion of complexity is just the ordinary one.

We now give the interpretation of the $<$-connective for the BCT- and the BCA-logic respectively. Notice that formulas containing free variables are interpreted as their universal closure. Truth in a model is defined in the usual, recursive way and we only discuss the $\prec$-connective. Since we are working in the standard model we avail ourselves to a substitutional interpretation of the quantifier, i.e. a universally quantified sentence is true iff all its instances are true.

Definition A.3 (Truth in a model: the $\prec$-connective)**.** *For all $A, B \in \mathcal{L}_{\mathcal{T}}$*

(i) $\qquad (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{T}} A \prec B \iff (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{T}} A \,\&\, (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{T}} B \,\&\, \mathsf{Com}_{\mathsf{T}}(A) < \mathsf{Com}_{\mathsf{T}}(B)$

(ii) $\qquad (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} A \prec B \iff (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} A \,\&\, (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} B \,\&\, \mathsf{Com}_{\mathsf{A}}(A) < \mathsf{Com}_{\mathsf{A}}(B)$

Under both interpretations of $\prec$ all theorems of BC will be true independently of the properties of Tr. The verification of the basic axioms of BC is straightforward and since modus ponens is the only rule of proof the claim follows trivially. For the remaining axioms we need Tr to be a Tarskian truth set.

Definition A.4 (Truth set for $\mathcal{L}$)**.** *We denote the Gödel number of a sentence Φ of $\mathcal{L}$ by $\#\Phi$ and set:*[55]

$$\mathsf{Tr} := \{\#\Phi : \mathcal{N} \models \Phi\}.$$

Proof of Proposition A.1. The proof is by an induction on the length of a proof. The induction step is trivial.The induction step is trivial. We discuss the base case.

(I) The truth of the axioms of BC follows immediately from the definition of $\mathsf{Com}_{\mathsf{T}}$ and $\mathsf{Com}_{\mathsf{A}}$ and Definition A.3.

(II) For the compositional axioms note that Tr is a Tarskian truth set. It is well known that Tr is a suitable model for the compositional axioms for the boolean connectives and the quantifiers. We discuss the axiom (T$\prec$). Since the axiom will be true according to $\models_{\mathsf{T}}$ and $\models_{\mathsf{A}}$ we do not distinguish between the two satisfaction relations nor the different notions of complexity.

$$\begin{aligned}
(\mathcal{N}, \mathsf{Tr}) \models \mathrm{T}[\ulcorner\Phi\urcorner \prec \ulcorner\Psi\urcorner] &\iff \#(\Phi \prec \Psi) \in \mathsf{Tr} \\
&\iff \mathcal{N} \models \Phi \prec \Psi \\
&\iff \mathcal{N} \models \Phi \,\&\, \mathcal{N} \models \Psi \,\&\, \mathsf{Com}(\Phi) < \mathsf{Com}(\Psi) \\
&\iff \#\Phi \in \mathsf{Tr} \,\&\, \#\Psi \in \mathsf{Tr} \,\&\, \mathsf{Com}(\Phi) < \mathsf{Com}(\Psi) \\
&\iff (\mathcal{N}, \mathsf{Tr}) \models \mathrm{T}\ulcorner\Phi\urcorner \,\&\, (\mathcal{N}, \mathsf{Tr}) \models \mathrm{T}\ulcorner\Psi\urcorner \,\&\, \mathsf{Com}(\mathrm{T}\ulcorner\Phi\urcorner) < \mathsf{Com}(\mathrm{T}\ulcorner\Psi\urcorner) \\
&\iff (\mathcal{N}, \mathsf{Tr}) \models \mathrm{T}\ulcorner\Phi\urcorner \prec \mathrm{T}\ulcorner\Psi\urcorner
\end{aligned}$$

(III) The Substitution axioms for the logical connectives and quantifiers: let ρ be a function that counts the number of embeddings of the $\prec$-connective in a formula A. We show the validity of the substitution axioms by an induction on the number of embeddings of $\prec$. We discuss the case of (Sub$_\forall$). As induction hypothesis we assume for all formulas A with $\rho(A) < m$

$$(\mathcal{N}, \mathsf{Tr}) \models A(\ulcorner\forall v\Phi\urcorner/C) \leftrightarrow A(\forall y \mathrm{T}\ulcorner\Phi\urcorner(\dot{y}/v)/C).$$

[55] The truth condition of a sentence $\Phi \prec \Psi$ are those of Definition A.3. Notice that $\mathsf{Com}_{\mathsf{T}}$ and $\mathsf{Com}_{\mathsf{A}}$ agree on sentences of $\mathcal{L}$.

Now let $\rho(A \prec B) = m$. We argue as follows

$$
\begin{aligned}
&(\mathcal{N}, \mathsf{Tr}) \models A(\mathrm{T}\ulcorner \forall v \Phi \urcorner / C) \prec B(\mathrm{T}\ulcorner \forall v \Phi \urcorner / C) \\
&\iff (\mathcal{N}, \mathsf{Tr}) \models A(\mathrm{T}\ulcorner \forall v \Phi \urcorner / C) \,\&\, (\mathcal{N}, \mathsf{Tr}) \models B(\mathrm{T}\ulcorner \forall v \Phi \urcorner / C) \,\&\, \mathsf{Com}(A(\mathrm{T}\ulcorner \forall v \Phi \urcorner)) < \mathsf{Com}(B(\mathrm{T}\ulcorner \forall v \Phi \urcorner)) \\
&\iff^{\dagger} (\mathcal{N}, \mathsf{Tr}) \models A(\forall y \mathrm{T}\ulcorner \Phi \urcorner (\dot{y}/v)) \,\&\, (\mathcal{N}, \mathsf{Tr}) \models B(\forall y \mathrm{T}\ulcorner \Phi \urcorner (\dot{y}/v)) \,\&\, \\
&\qquad \mathsf{Com}(A(\forall y \mathrm{T}\ulcorner \Phi \urcorner (\dot{y}/v)) < \mathsf{Com}(B(\forall y \mathrm{T}\ulcorner \Phi \urcorner (\dot{y}/v))) \\
&\iff (\mathcal{N}, \mathsf{Tr}) \models A(\forall y \mathrm{T}\ulcorner \Phi \urcorner (\dot{y}/v)/C) \prec B((\forall y \mathrm{T}\ulcorner \Phi \urcorner (\dot{y}/v)/C)
\end{aligned}
$$

The †-equivalence follows (i) from the induction hypothesis, (ii) from the definition of $\mathsf{Com}_{\mathsf{T}}$ and $\mathsf{Com}_{\mathsf{A}}$, and (iii) the fact that the set $\{\mathsf{Com}(\mathrm{T}\ulcorner \Phi \urcorner (\dot{t}/v)) \,:\, t \in \mathsf{Term}_{\mathcal{L}_{\mathrm{T}}}\}$ always has a maximum:

$$
\begin{aligned}
\mathsf{Com}_{\mathsf{A}}(\forall y \mathrm{T}\ulcorner \Phi \urcorner (\dot{y}/v)) &= \mathsf{Sup}(\{\mathsf{Com}_{\mathsf{A}}(\mathrm{T}\ulcorner \Phi \urcorner (\dot{t}/v)) \,:\, t \in \mathsf{Term}_{\mathcal{L}_{\mathrm{T}}}\}) + 1 \\
&= \mathsf{Sup}(\{\mathsf{Com}_{\mathsf{A}}(\mu(\mathrm{T}\ulcorner \Phi \urcorner (\dot{t}/v))) + 1 \,:\, t \in \mathsf{Term}_{\mathcal{L}_{\mathrm{T}}}\}) + 1 \\
&= \mathsf{Sup}(\{\mathsf{Com}_{\mathsf{A}}(\Phi(t/v)) + 1 \,:\, t \in \mathsf{Term}_{\mathcal{L}_{\mathrm{T}}}\}) + 1 \\
&= \mathsf{Max}(\{\mathsf{Com}_{\mathsf{A}}(\Phi(t/v)) + 1 \,:\, t \in \mathsf{Term}_{\mathcal{L}_{\mathrm{T}}}\}) + 1 \\
&= \mathsf{Max}(\{\mathsf{Com}_{\mathsf{A}}(\Phi(t/v)) \,:\, t \in \mathsf{Term}_{\mathcal{L}_{\mathrm{T}}}\}) + 2 \\
&= \mathsf{Com}_{\mathsf{A}}(\forall v \Phi) + 1 \\
&= \mathsf{Com}_{\mathsf{A}}(\mu(T\ulcorner \forall v \Phi \urcorner)) + 1 \\
&= \mathsf{Com}_{\mathsf{A}}(T\ulcorner \forall v \Phi \urcorner).
\end{aligned}
$$

The case for $\mathsf{Com}_{\mathsf{T}}$ is very much parallel.

(IV) We now focus on $\models_{\mathsf{T}}$ and show the validity of ($\mathrm{Sub}_{\mathrm{At}}$). This again by an induction on the number of embeddings of $\prec$. As induction hypothesis we assume for all formulas A with $\rho(A) < m$

$$(\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{T}} A(\mathrm{T}\ulcorner s = t \urcorner / C) \leftrightarrow A(s = t/C).$$

The argument is parallel to the one displayed in (III). We note that by definition

$$\mathsf{Com}_{\mathsf{T}}(\mathrm{T}\ulcorner s = t \urcorner) = \mathsf{Com}_{\mathsf{T}}(\mu(\mathrm{T}\ulcorner s = t \urcorner)) = \mathsf{Com}_{\mathsf{T}}(s = t).$$

(V) Turning to $\models_{\mathsf{A}}$ we now show the validity ($\prec$T$\downarrow$). First notice that if $A \notin \mathsf{Sent}_{\mathcal{L}}$ then there is nothing to show for the antecedent of the conditional will be false. We thus assume ($*$) $(\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} \mathrm{T}\ulcorner \Phi \urcorner$ and need to show $(\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} \Phi \prec \mathrm{T}\ulcorner \Phi \urcorner$. From ($*$) we infer $\#\Phi \in \mathsf{Tr}$ and hence $\mathcal{N} \models \Phi$ but also $(\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} \Phi$. Moreover, we have

$$\mathsf{Com}_{\mathsf{A}}(\mathrm{T}\ulcorner \Phi \urcorner) = \mathsf{Com}_{\mathsf{A}}(\mu(\mathrm{T}\ulcorner \Phi \urcorner)) + 1 = \mathsf{Com}_{\mathsf{A}}(\Phi) + 1$$

and hence $\mathsf{Com}_{\mathsf{A}}(\Phi) < \mathsf{Com}_{\mathsf{A}}(\mathrm{T}\ulcorner \Phi \urcorner)$. That is, we have

$$(\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} \Phi \,\&\, (\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} \mathrm{T}\ulcorner \Phi \urcorner \,\&\, \mathsf{Com}_{\mathsf{A}}(\Phi) < \mathsf{Com}_{\mathsf{A}}(\mathrm{T}\ulcorner \Phi \urcorner).$$

By Definition A.3 this implies $(\mathcal{N}, \mathsf{Tr}) \models_{\mathsf{A}} \Phi \prec \mathrm{T}\ulcorner \Phi \urcorner$.

□